CONJUGATE PROBLEM OF HEAT TRANSFER IN A LAMINAR BOUNDARY LAYER WITH INJECTION

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The author examines two conjugate problems of heat transfer in the laminar boundary layer at the boundary of a semi-infinite porous medium on the assumption that fluid filters continuously through the porous surface and that the injection velocity varies as $x^{-1/2}$, where x is the distance in the direction of flow.

The problem of heat transfer in a boundary layer with injection consists in solving the ordinary boundary layer equations with boundary conditions imposed on the transverse velocity component at the surface, the diffusion equation for the injected gas, and the modified energy equation. This problem has been examined by various authors [1–3]. Certain numerical solutions were obtained by Eckert et al. [4]. Eckert and Drake [5] investigated the problem for an injection velocity varying in inverse proportion to the square root of the distance from the leading edge for Pr = Sc = = 1 and Pr = Sc = 0.7; where Pr and Sc are the Prandtl and Schmidt numbers, respectively.

However, all these authors investigated the problem for a given temperature along the surface, completely neglecting the heat conduction of the porous body. Their solutions do not depend on the thermophysical characteristics of the porous surface. In this paper, we examine two conjugate problems of heat transfer in a laminar boundary layer with injection of the same type as that considered by Eckert and Drake [5], taking into account the thermal conductivity of the porous medium at whose surface the boundary layer exists. Thus, the solutions depend on the thermophysical characteristics of the porous medium. Conjugate heat-transfer problems were first examined by Perel'man [6, 7], who solved the boundary layer equations together with the equations of heat conduction in the solid on the assumption of continuity of the temperature and heat flux at the surface.

The first problem considered in this paper is concerned with the boundary layer at the boundary of a semi-infinite porous medium, $0 < x < \infty$, $-\infty < y < 0$. It is assumed that the injected fluid continuously filters through the surface of the porous body and is instantaneously evaporated at the surface, absorbing the heat of evaporation and thus cooling the system. Heat conduction in the direction of the main stream is neglected, but convective heat transfer in the porous body due to the motion of the fluid is taken into account. This corresponds to the case when the injection velocity is not very small in comparison with the main stream velocity.

In the second problem, we neglect convective heat transport in the porous body, but take heat conduction in the longitudinal direction into account.

Whereas the first problem reduces to the solution of a singular integral equation for the temperature at the surface for which it is possible to obtain an exact solution, the second reduces to the joint solution of two singular integral equations, for which asymptotic solutions at large values of the distance in the direction of flow are obtained by the method proposed in [8].

Problem 1. The equations describing the problem are as follows (see figure):

for the velocity field

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}, \tag{1}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2}$$

$$u|_{y=0} = 0, \tag{3}$$

$$v|_{y=0} = -\frac{C}{2}\sqrt{\frac{\nu U_\infty}{x}} = v_0(x), \tag{4}$$

$$u|_{y\to\infty} = U_\infty, \tag{5}$$

where C is found from (4) as

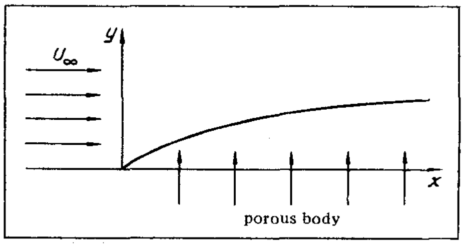
$$C = C_q \sqrt{\frac{U_\infty l}{\nu}}, \tag{6}$$

$$C_q = \frac{-q}{lbU_\infty}; \tag{7}$$

for the temperature field: fluid—

$$u \frac{\partial \theta}{\partial y} + v \frac{\partial \theta}{\partial y} = \frac{\nu}{\sigma} \frac{\partial^2 \theta}{\partial y^2}, \quad 0 < x < \infty,$$

$$0 < y < \infty, \tag{8}$$



Boundary layer on a porous body

porous body—

$$\frac{\partial^2 T}{\partial y^2} = \frac{v_0(x)}{\alpha_s}\frac{\partial T}{\partial y} - \frac{Q(x, y)}{k_s}, \quad 0 < x < \infty,$$

$$-\infty < y < 0. \tag{9}$$

To be specific, we assume that

$$Q(x, y) = \begin{cases} Q(x), & -h < y < 0, \\ 0, & -\infty < y < -h. \end{cases} \tag{10}$$

The boundary conditions are

$$\theta = T = \Theta(x), \quad y = 0; \tag{11}$$

$$k_f \frac{\partial \theta}{\partial y} = k_s \frac{\partial T}{\partial y} - \frac{C}{2}\rho_f \sqrt{\frac{\nu U_\infty}{x}} I = p(x), \quad y = 0; \tag{12}$$

$$\theta = 0, \quad x = 0; \tag{13}$$

$$\theta = 0, \quad y \to \infty; \tag{14}$$

$$\left.\frac{\partial T}{\partial y}\right|_{y\to-\infty} = 0. \tag{15}$$

The solution of Eqs. (1) and (2) with the boundary conditions (3), (4), and (5) was obtained by Schlichting and Bussmank [9] for various values of C. Using the transformation

$$\eta = \frac{1}{2} y \sqrt{\frac{U_\infty}{\nu x}} \tag{16}$$

we reduce Eqs. (1) and (2) to the form

$$ff'' + f''' = 0 \tag{17}$$

with the boundary conditions

$$f' = 0, \quad f = C, \quad \eta = 0, \tag{18}$$

$$f' \to 2, \quad \eta \to \infty. \tag{19}$$

Thus, the surface friction is given by the formula

$$\tau_0 = \mu\left(\frac{\partial U}{\partial y}\right)_{y=0} = \mu U_\infty \sqrt{\frac{U_\infty}{\nu x}}\frac{1}{4} f''(0) =$$

$$= \mu U_\infty \sqrt{\frac{U_\infty}{\nu x}} K(C). \tag{20}$$

The value of $(1/4)f''(0) = K(C)$ can be obtained from [9], in which tables were compiled for the dependence of $f''(0)$ on various values of C and $K(0) = 0.332$ for the Blasius boundary layer.

The solution of Eq. (8) with the boundary conditions (13) and (14) and the surface friction (20) can be obtained by the Lighthill method [10]. Thus, the relation between $\Theta(x)$ and the flow at the surface $p(x)$ is determined from Eqs. (11) and (12). Therefore,

$$p(x) = -K(C)\frac{k_f}{\mu}\sigma^{1/3}\left(\frac{\mu\rho_f U_\infty}{x}\right)^{1/2} \times$$

$$\times \left\{\Theta(x) + x^{1/4}\int_0^x \frac{\Theta'(x_1)\,dx_1}{(x^{3/4} - x_1^{3/4})^{1/3}}\right\}. \tag{21}$$

The solution of Eq. (9) with the boundary conditions (15) is as follows:

$$T(x, y) = -\int_{-\infty}^{y} \exp\left[\frac{v_0(x)\,y}{\alpha_s}\right]\left[\int_0^y \frac{Q(x, y')}{k_s} \times\right.$$

$$\left.\times \exp\left[\frac{-v_0(x)\,y'}{\alpha_s}\right] dy'\right] dy +$$

$$+ \frac{p^{(1)}(x)}{k_s}\int_{-\infty}^{y} \exp\left(\frac{v_0(x)\,y}{\alpha_s}\right) dy, \tag{22}$$

where

$$p^{(1)}(x) = p(x) + \frac{C}{2}\rho_f \sqrt{\frac{\nu U_\infty}{x}} I, \tag{23}$$

and, thus, from condition (11) we obtain the expression

$$\Theta(x) = -\int_{-\infty}^{0} \exp\left[\frac{v_0(x)\,y}{\alpha_s}\right]\left[\int_0^y \frac{Q(x, y')}{k_s} \times\right.$$

$$\left.\times \exp\left[-\frac{v_0(x)\,y'}{\alpha_s}\right] dy'\right] dy +$$

$$+ \frac{p^{(1)}(x)}{k_s}\int_{-\infty}^{0} \exp\left[\frac{v_0(x)\,y}{\alpha_s}\right] dy, \tag{24}$$

which, together with (10), leads to

$$\Theta(x) = \frac{-2\alpha_s\sqrt{x}}{Ck_s\sqrt{\nu U_\infty}}\{hQ(x) + p(x)\} - \left\{\frac{I}{c_s}\frac{\rho_f}{\rho_s}\right\}. \tag{25}$$

Eliminating p(x) from (25) and (21), we arrive at the following singular integral equation for $\Theta(x)$:

$$\Theta(x)(\beta - 1) + \beta x^{1/4}\int_0^x \frac{\theta'(x_1)\,dx_1}{(x^{3/4} - x_1^{3/4})^{1/3}} = \frac{2\alpha_s\sqrt{x}\,hQ(x)}{Ck_s\sqrt{\nu U_\infty}}, \tag{26}$$

where

$$\beta = \frac{2K(C)\,\alpha'\sigma^{-2/3}}{C\chi}; \tag{27}$$

$$\chi = k_s/k_f; \tag{28}$$

$$\alpha' = \alpha_s/\alpha_f. \tag{29}$$

Case 1. Constant source

$$Q(x) = \begin{cases} Q_0, & x < L, \\ 0, & x > L. \end{cases} \tag{30}$$

Equation (26) can be represented in the following form:

$$\Theta(x) + \gamma x^{1/4}\int_0^x \frac{\Theta'(x)_1\,dx_1}{(x^{3/4} - x_1^{3/4})^{1/4}} = H\sqrt{x} + \frac{I\rho_f}{c_s\rho_s}, \tag{31}$$

where

$$\gamma = \frac{\beta}{\beta - 1}; \tag{32}$$

$$H = \begin{cases} \dfrac{2\alpha_s hQ_0}{Ck_s\sqrt{\nu U_\infty}}, & x < L, \\ 0, & x > L. \end{cases} \tag{33}$$

Taking the Mellin transform of Eq. (31)

$$\Theta(S)=\int_0^\infty \Theta(x)\,x^{S-1}\,dx, \tag{34}$$

we obtain

$$\Theta(S)\left\{1+\gamma\frac{\Gamma\left(1-\frac{4}{3}S\right)\Gamma\left(\frac{2}{3}\right)}{\Gamma\left(\frac{2}{3}-\frac{4}{3}S\right)}\right\}=\frac{HL^{S+1/2}}{S+\frac{1}{2}}+$$
$$+\left(\frac{I\rho_f}{c_s\rho_s}\right)\varepsilon^{-S}\Gamma(S), \tag{35}$$

where, so that we can apply the Mellin transformation, the second term on the right-hand side of Eq. (31) has been represented in the form [7]

$$\left(\frac{I}{c_s}\frac{\rho_f}{\rho_s}\right)=\underset{\varepsilon\to 0}{e^{-\varepsilon x}}\left(\frac{I}{c_s}\frac{\rho_f}{\rho_s}\right), \tag{36}$$

and, thus,

$$\Theta(S)=\left[HL^{S+1/2}\,\Gamma\left(\frac{2}{3}-\frac{4}{3}S\right)\right]\times$$
$$\times\left[(S+1/2)\left\{\Gamma\left(\frac{2}{3}-\frac{4}{3}S\right)+\right.\right.$$
$$\left.\left.+\gamma\Gamma\left(\frac{2}{3}\right)\Gamma\left(1-\frac{4}{3}S\right)\right\}\right]^{-1}+$$
$$+\frac{\left(\frac{I\rho_f}{c_s\rho_s}\right)\varepsilon^{-S}\,\Gamma(S)\,\Gamma\left(\frac{2}{3}-\frac{4}{3}S\right)\Bigg|_{\varepsilon\to 0}}{\left\{\Gamma\left(\frac{2}{3}-\frac{4}{3}S\right)+\gamma\Gamma\left(\frac{2}{3}\right)\Gamma\left(1-\frac{4}{3}S\right)\right\}}. \tag{37}$$

The inverse Mellin transformation can now be carried out without difficulty, since, in the second term, the contribution of all the singularities vanishes as $\varepsilon\to 0$, apart from the case S = 0. Thus,

$$\Theta(x)=\begin{cases}\dfrac{Hx^{1/2}}{\delta\left(-\frac{1}{2}\right)}+\left(\dfrac{I\rho_f}{c_s\rho_s}\right)\Big/\delta(0), & 0<x<L,\\[2ex] \left(\dfrac{I\rho_f}{c_s\rho_s}\right)\Big/\delta(0), & x>L,\end{cases} \tag{38}$$

where

$$\delta\left(-\frac{1}{2}\right)=1+\frac{2\gamma\left(\Gamma\left(\frac{2}{3}\right)\right)^2}{\Gamma\left(\frac{1}{3}\right)} \tag{39}$$

and

$$\delta(0)=1+\gamma. \tag{40}$$

Thus, the temperature varies as $x^{1/2}$ in the presence of constant sources in the porous medium, while, in the absence of sources, a constant temperature is maintained.

Similarly,

$$p(x)=-\left\{\frac{Hx^{1/2}}{\delta\left(-\frac{1}{2}\right)}+\frac{\left(\frac{I}{c_s}\frac{\rho_f}{\rho_s}\right)\gamma}{1+\gamma}\right\}\times$$
$$\times\frac{C\rho_s c_s}{2}\sqrt{\frac{\nu U_\infty}{x}}-hQ_0,\quad 0<x<L,\text{or}$$

$$p(x)=-\frac{\gamma}{1+\gamma}\left[I\rho_f v_0(x)\right],\qquad x>L. \tag{41}$$

Thus, for x > L the heat-transfer coefficient

$$\alpha^*=\frac{p(x)}{\Theta(x)}=\frac{\beta}{2(\beta-1)}Cc_s\rho_s\sqrt{\frac{\nu U_\infty}{x}} \tag{42}$$

and the local Nusselt number

$$\mathrm{Nu}_x=\frac{\beta}{2(\beta-1)}Cc\rho_s\sqrt{\nu U_\infty x}, \tag{43}$$

where β is determined from Eq. (27).

Case 2. $Q(x)=\sqrt{h}Q_0/\sqrt{x}$. Proceeding as in case 1, we obtain

$$\Theta(x)=\frac{H'}{\delta(0)}=\text{const} \tag{44}$$

and

$$p(x)=-\left\{\frac{H'}{\delta(0)}+\frac{I\rho_f}{c_s\rho_s}\right\}\frac{Ck_s}{2a_s}\sqrt{\frac{\nu U_\infty}{x}}-\frac{h^{3/2}Q_0}{\sqrt{x}}, \tag{45}$$

where

$$H'=\frac{2a_s h^{3/2}Q_0}{Ck_s\sqrt{\nu U_\infty}}+\left(\frac{I\rho_f}{c_s\rho_s}\right). \tag{46}$$

Thus, in this case,

$$\alpha^*=-\left\{1+\frac{I\rho_f\delta(0)}{H'c_s\rho_s}\right\}\frac{Ck_s}{2a_s}\sqrt{\frac{\nu U_\infty}{x}}-\frac{h^{3/2}Q_0\,\delta(0)}{H'\sqrt{x}} \tag{47}$$

and

$$\mathrm{Nu}_x=\left\{1+\frac{I\rho_f\,\delta(0)}{H'c_s\rho_s}\right\}\frac{C\chi}{2a_s}\sqrt{\nu U_\infty x}-\frac{h^{3/2}Q_0\,\delta(0)\sqrt{x}}{H'k_f}. \tag{48}$$

Problem 2. We now consider the analogous problem in which we neglect convective heat transport in the porous body and take the longitudinal heat conduction into account. Thus, we must solve the system of equations (1)–(15), replacing Eq. (9) with the equation

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=0 \tag{49}$$

with the additional boundary conditions

$$T=0,\quad x=0, \tag{50}$$

$$\left.\frac{\partial T}{\partial x}\right|_{x\to\infty}=0. \tag{51}$$

The solution of Eq. (49), satisfying conditions (15), (50), and (51), has the form [7]

$$T(x,\ y)=\frac{1}{2\pi}\int_0^\infty \ln\frac{(x+x')^2+y^2}{(x-x')^2+y^2}\,p^{(1)}(x')\,dx'. \tag{52}$$

Thus, from (11) and (12) we obtain

$$\Theta(x)=\frac{1}{\pi}\int_0^\infty \ln\left|\frac{x+y}{x-y}\right|p^{(1)}(y)\,dy, \tag{53}$$

where

$$p^{(1)}(x)=\left.\frac{\partial T}{\partial y}\right|_{y=0}=\frac{1}{k_s}p(x)+\frac{\gamma'}{\sqrt{x}}, \tag{54}$$

and p(x) is determined from (11) and (12) and

$$\gamma' = \frac{C}{2}\frac{\rho_f I}{k_s}\sqrt{\nu U_\infty}. \tag{55}$$

Function p(x) is again determined in terms of Θ using Eq. (21). Thus, the solution of the problem reduces to the joint solution of two singular integral equations, (53) and (21), with account for (54).

Taking the Mellin transform of functions (53), (54), and (21), we obtain

$$\Theta(S) = \frac{p^{(1)}(S+1)}{S}\operatorname{tg}\left(\frac{\pi}{2}S\right), \quad -1 < \operatorname{Re} S < 1, \tag{56}$$

$$p^{(1)}(S) = \frac{1}{k_s}p(S) + \gamma' \underset{\varepsilon\to 0}{\varepsilon^{S-\frac{1}{2}}}\Gamma\left(\frac{1}{2}-S\right) \tag{57}$$

and

$$p(S) = -\beta'\Theta\left(S-\frac{1}{2}\right)\times$$

$$\times\left[1+\frac{\Gamma\left(\frac{5}{3}-\frac{4}{3}S\right)\Gamma\left(\frac{2}{3}\right)}{\Gamma\left(\frac{4}{3}-\frac{4}{3}S\right)}\right],$$

$$0<\operatorname{Re} S<1/4, \tag{58}$$

where, to obtain the Mellin transform, the second term on the right-hand side of (54) was written in the form

$$\frac{\gamma'}{\sqrt{x}} = \underset{\varepsilon\to 0}{e^{-\varepsilon/x}}\frac{\gamma'}{\sqrt{x}} \tag{59}$$

and β' in (58) is expressed as

$$\beta' = K(C)\,k_f\,\sigma^{1/3}\left(\frac{\rho_f U_\infty}{\mu}\right)^{1/2}. \tag{60}$$

Equation (56) can be written as follows:

$$\Theta(S-1) = \frac{p^{(1)}(S)}{S-1}\operatorname{tg}\frac{\pi}{2}(S-1), \quad 0<\operatorname{Re} S<2. \tag{61}$$

Eliminating $p^{(1)}(S)$ and p(S) from Eqs. (57), (58), and (61), we obtain the following difference equation in the Mellin transform of the function $\Theta(x)$:

$$\Theta(S-1) = -\frac{\beta'}{k_s}\frac{\Theta(S-1/2)}{S-1}\times$$

$$\times\left[1+\frac{\Gamma\left(\frac{5}{3}-\frac{4}{3}S\right)\Gamma\left(\frac{2}{3}\right)}{\Gamma\left(\frac{4}{3}-\frac{4}{3}S\right)}\right]\times$$

$$\times\operatorname{tg}\frac{\pi}{2}(S-1) + \gamma' \underset{\varepsilon\to 0}{\varepsilon^{S-1/2}}\times$$

$$\times\frac{\Gamma\left(\frac{1}{2}-S\right)\operatorname{tg}\frac{\pi}{2}(S-1)}{S-1}. \tag{62}$$

We now find the asymptotic solution of this equation for large x. In (62) we replace $\operatorname{tg}(\pi/2)(S-1)$ in the first term on the right-hand side by the equivalent expression using Γ functions.

Thus,

$$\Theta(S-1) = -\frac{\beta}{k_s}\frac{\Theta(S-1/2)}{S-1}\times$$

$$\times\left[1+\frac{\Gamma\left(\frac{5}{3}-\frac{4}{3}S\right)\Gamma\left(\frac{2}{3}\right)}{\Gamma\left(\frac{4}{3}-\frac{4}{3}S\right)}\right]\times$$

$$\times\left[\frac{\Gamma(1-S/2)\,\Gamma(S/2)}{\Gamma\left(\frac{3}{2}-\frac{S}{2}\right)\Gamma\left(\frac{1}{2}+S/2\right)} - \gamma' \underset{\varepsilon\to 0}{\varepsilon^{S-1/2}}\Gamma\times\right.$$

$$\left.\times\left(\frac{1}{2}-S\right)\frac{\operatorname{tg}\frac{\pi}{2}(S-1)}{S-1}\right],$$

$$0<\operatorname{Re} S<1/4. \tag{63}$$

It should be noted that the first term on the right-hand side of Eq. (63) was obtained for $0 < \operatorname{Re} S < 1/4$; the second term is valid for the region $0 < \operatorname{Re} S < 2$. However, if as in [8], we set

$$\Theta\left(\frac{3}{4}S-1\right) = \Omega(S)\,\psi(S), \quad \operatorname{Re} S>0, \tag{64}$$

where

$$\Omega(S) = \left[\Gamma(2,S)\,\Gamma\left(S+\frac{4}{3}\right)\times\right.$$

$$\times\Gamma\left(S+\frac{1}{3}\right)\Gamma(S+1)\,\Gamma\left(\frac{3}{8}S+\right.$$

$$\left.\left.+\frac{5}{4}\right)\Gamma\left(\frac{3}{4}S+\frac{1}{2}\right)\Gamma\left(\frac{3}{8}S+\frac{1}{4}\right)\frac{3}{4}(S-2)\right]\times$$

$$\times\left[\Gamma\left(\frac{7}{3}-S\right)\Gamma\left(\frac{5}{4}-\frac{3}{8}S\right)\right]^{-1},$$

$$\operatorname{Re} S>0; \tag{65}$$

$$\psi(x) \simeq \sum_{n=0}^{\infty}\frac{a_n}{x^{\alpha n+\delta}}, \tag{66}$$

we can easily show that the inverse Mellin transformation for $\Omega(S)$ exists for all x, including $x\to\infty$, and $\Omega(S)$ does not have singularities in the half-plane $\operatorname{Re} S > 0$.

Substituting (64) into (63), we note that, for the second term on the right-hand side of the equation, the contribution of all the other poles $\to 0$ as $\varepsilon\to 0$, and only the pole $S = 1/2$ gives -2γ. Assumption (66) relative to $\psi(x)$ leads to values of the coefficients $\delta = 1$, $\alpha = 1/2$, and we obtain

$$\Theta(x) = \frac{3}{4}\sum_{n=0}^{\infty}\frac{\Omega\left(\frac{2n}{3}+\frac{4}{3}\right)a_n}{x^{n/2}} \tag{67}$$

and from (58)

$$p(x) = -\frac{3}{4}\beta'\sum_{n=0}^{\infty}\frac{K_1\left(\frac{2n}{3}+\frac{4}{3}\right)a_n}{x^{n/2+1/2}}, \tag{68}$$

where

$$K_1(S) = K_2(S)\left[1+\frac{\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{7}{3}-S\right)}{\Gamma(2-S)}\right], \tag{69}$$

and

$$K_2(S) = \left[\Gamma\left(2S + \frac{4}{3}\right)\Gamma(S+2)\Gamma(S+1)\times\right.$$

$$\times\Gamma\left(S + \frac{5}{3}\right)\Gamma\left(\frac{3}{8}S + 3/2\right)\times$$

$$\left.\times\Gamma\left(\frac{3}{4}S + 1\right)\Gamma\left(\frac{3}{8}S\right)\right]\times$$

$$\times\left[\Gamma\left(\frac{4}{3} - S\right)\Gamma\left(\frac{3}{2} - \frac{3}{8}S\right)\right]^{-1}, \tag{70}$$

The coefficients a_n, $n > 0$ are determined by means of the recurrence relations

$$\Omega\left(\frac{2n}{3} + \frac{2}{3}\right)a_{n-1} = -\frac{\beta'}{k_s}K\left(\frac{2n}{3} + \frac{2}{3}\right)a_n,$$

$$a_n = 0, \ n < 0 \tag{71}$$

and

$$a_0 = -\frac{2k_s\gamma'}{\beta'}\frac{1}{K_1(2/3)}. \tag{72}$$

From (60)

$$a_0 = -\frac{2\chi\gamma'}{K(C)}\sigma^{-1/3}\left(\frac{\nu}{U_\infty}\right)^{1/2}\frac{1}{K_1(2/3)} \tag{73}$$

and

$$a_1 = 2\frac{\chi^2\gamma'}{K^2(C)}\sigma^{-2/3}\times$$

$$\times\left(\frac{\nu}{U_\infty}\right)\frac{1}{K_1\left(\frac{2}{3}\right)}\frac{\Omega\left(\frac{4}{3}\right)}{K_1\left(\frac{4}{3}\right)},$$

$$a_2 = -2\frac{\chi^3\gamma'}{K^3(C)}\sigma^{-1}\left(\frac{\nu}{U_\infty}\right)^{3/2}\times$$

$$\times\frac{\Omega\left(\frac{4}{3}\right)\Omega(2)}{K_1\left(\frac{2}{3}\right)K_1\left(\frac{4}{3}\right)K_1(2)}.$$

Thus,

$$\Theta(x) = -\frac{3}{2}x^{1/2}\gamma'\left[\frac{\chi}{K(C)}\sigma^{-1/3}\frac{(\mathrm{Re}_x)^{-1/2}\Omega\left(\frac{4}{3}\right)}{K_1\left(\frac{2}{3}\right)} - \right.$$

$$-\frac{\chi^2}{K^2(C)}\sigma^{-2/3}\frac{(\mathrm{Re}_x)^{-1}\Omega\left(\frac{4}{3}\right)\Omega(2)}{K_1\left(\frac{2}{3}\right)K_1\left(\frac{4}{3}\right)} +$$

$$+\frac{\chi^3}{K^3(C)}\sigma^{-1}\times$$

$$\left.\times\frac{(\mathrm{Re}_x)^{-3/2}\Omega\left(\frac{4}{3}\right)\Omega(2)\Omega\left(\frac{8}{3}\right)}{K_1\left(\frac{2}{3}\right)K_1\left(\frac{4}{3}\right)K_1(2)} - \ldots\right] \tag{74}$$

and

$$p(x) = \frac{3}{2}\gamma' k_s\left[\frac{K_1\left(\frac{4}{3}\right)}{K_1\left(\frac{2}{3}\right)} - \frac{\chi}{K(C)}\sigma^{-1/3}(\mathrm{Re}_x)^{-1/2}\times\right.$$

$$\times\frac{\Omega\left(\frac{4}{3}\right)K_1(2)}{K_1\left(\frac{2}{3}\right)K_1\left(\frac{4}{3}\right)} +$$

$$+\frac{\chi^2}{K^2(C)}\sigma^{-2/3}(\mathrm{Re}_x)^{-3/2}\times$$

$$\left.\times\frac{\Omega\left(\frac{4}{3}\right)\Omega(2)K_1\left(\frac{8}{3}\right)}{K_1\left(\frac{2}{3}\right)K_1\left(\frac{4}{3}\right)K_1(2)} - \ldots\right]. \tag{75}$$

Thus, since $\Omega(2) = 0$ from Eq. (65), it follows from (71) that all the a_n with $n > 2$ vanish and

$$\Theta(x) = -\frac{3}{2}x^{1/2}\gamma'\frac{\chi}{K(C)}\sigma^{-1/3}(\mathrm{Re}_x)^{-1/2}\times$$

$$\times\frac{\Omega\left(\frac{4}{3}\right)}{K_1\left(\frac{2}{3}\right)} = \mathrm{const}, \tag{76}$$

while

$$p(x) = \frac{3}{2}\gamma' k_s\left[\frac{K_1\left(\frac{4}{3}\right)}{K_1\left(\frac{2}{3}\right)} - \right.$$

$$\left.-\frac{\chi}{K(C)}\sigma^{-1/3}(\mathrm{Re}_x)^{-1/2}\frac{\Omega\left(\frac{4}{3}\right)K_1(2)}{K_1\left(\frac{2}{3}\right)K_1\left(\frac{4}{3}\right)}\right]. \tag{77}$$

Consequently,

$$\alpha^* = -\frac{k_s\chi}{K(C)}\sigma^{-1/3}\sqrt{\frac{\nu}{U_\infty}}\left[\frac{K_1\left(\frac{4}{3}\right)}{\Omega\left(\frac{4}{3}\right)} - \right.$$

$$\left.-\frac{\chi}{K(C)}\sigma^{-1/3}(\mathrm{Re}_x)^{-\frac{1}{2}}\frac{K_1(2)}{K_1\left(\frac{4}{3}\right)}\right], \tag{78}$$

Thus, for large values of x, the temperature becomes constant and the heat flux varies as $x^{-1/2}$; the asymptotic solution gives a result qualitatively similar to the result obtained from problem 1 in the absence of sources in the body. This is because, at large distances, it is possible to neglect the conductivity of the body in the direction of flow. The constancy of the temperature as the injection velocity varies along the plate according to the law $x^{-1/2}$ has been noted before, for example, in [5], where it was concluded that the assumption of such a law of variation of injection velocity is equivalent to the assumption that the surface is at constant temperature and, hence, that the assumption of constant temperature and variation of injection velocity according to the law $x^{-1/2}$ was justified for the solution of the nonconjugate problem.

NOTATION

u is the fluid velocity in the x-direction; v is the fluid velocity in the y-direction; $v_0(x)$ is the injection velocity at the surface of the porous body; ν is the kinematic coefficient of viscosity; q is the mass of fluid filtering through a small area of porous surface of length l and width b per unit time; μ is the dynamic coefficient of viscosity; θ is the fluid temperature; T is the temperature of the porous body; k is the thermal conductivity; ρ is the density; c is the specific heat; $\alpha = k/\rho c$ is the thermal diffusivity; I is the specific heat of evaporation. Subscripts: s relates to the porous body, f to the fluid.

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